

Communication Theory Homework 1

Professor: Brian Frost

Fall 2020

1. A square wave has period T , amplitude A and duty cycle τ/T (the signal takes value A from time 0 to time τ , then 0 from time τ to T). Find the Fourier series representation of this signal.
2. Does the signal from the previous question have finite energy? Does it have finite power? If the answer to either question is yes, find the value of the energy or power.
3. What is $\text{sinc}(t) * \text{sinc}(t)$? **Hint:** use the convolution theorem.
4. Suppose a system acts on signal x and produces output y by the rule

$$y(t) = |x(t + 3)|.$$

Is the system linear? Is the system time invariant? Is the system causal?

5. Let $x(t)$ be the signal given by $\cos(2\pi ft)$ for $0 \leq t \leq T$ and 0 otherwise, where $T = 1/f$. Find the Fourier transform, $X(\omega)$, of this signal. Demonstrate Parseval's theorem by comparing the norms of x and X .
6. Suppose we pass $x(t)$ from the previous question through the system from question 4. Use MATLAB to find the amplitude and phase of the output signal's Fourier transform $Y(\omega)$. Plot $X(\omega)$ as well – how do the signals compare?
7. Let $z(t) = y(t) \cos(64\pi t + \theta)$. Write z as a sum of in-phase and quadrature components. Plot the Fourier transform $Z(\omega)$ for $\theta = \pi/3$, and comment on the effect of the modulation on the amplitude and phase (comparing to $Y(\omega)$).
8. Write a function that takes as an input a time-domain signal and outputs the Hilbert transform of that signal (also in the time-domain). Plot the Hilbert transforms of x , y and z in the frequency and time domains.
9. Using MATLAB, demonstrate the orthogonality of a signal and its Hilbert transform for all of x , y and z .