Communication Theory Homework 3

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1. A source generates 0s and 1s randomly according to a Bernoulli distribution, where the probability of a 0 is 0.3 and the probability of a 1 is 0.7. The value is then sent across a long wire, and corrupted by thermal noise. A system at the other end receives the corrupted signal and guesses if it received a 1 or a 0. It has an error probability (either guessing a 1 was a 0 or guessing a 0 was a 1) of 0.2. If the receiver guesses it received a 1, what is the probability a 1 was transmitted?

2. Let Θ be a random variable uniformly distributed from 0 to π. Let $X = \cos\Theta$ and $Y = \sin\Theta$. Are X and Y uncorrelated? Are they independent? Are they orthogonal?

3. Let $X(t)$ be a random process defined by $X(t) = A + Bt$ where A and B are independent random variables uniformly distributed from -1 to 1. Find $m_X(t)$ and $R_X(t_1, t_2)$. Is the process WSS? If not, is it cyclostationary? If the answer to either question is yes, find the PSD of X.

4. Let $X(t) = Y\cos(\omega_0 t) - Z\sin(\omega_0 t)$, where Y and Z are zero-mean independent Gaussians with variance σ^2. Find $m_X(t)$ and $R_X(t_1, t_2)$. Is the process WSS? If not, is it cyclostationary? If the answer to either question is yes, find the PSD of X.

5. If $X(t)$ has PSD $S_X(\omega)$, what is the PSD of $4X'(t) + X(t-T)$?

6. In the discrete-time case, suppose we have a random process defined by $\{X_n\}$ for $n \in \mathbb{Z}$. If we observe N samples of the random process, $\{x_n\}$ for $n = 1, 2, \ldots, N$, we define the *sample autocorrelation* as

$$R_X(m) = \begin{cases} \frac{1}{N-m}\sum_{n=1}^{N-m} x_n x_{n+m}, & m = 0, 1, \ldots \\ \frac{1}{N-|m|}\sum_{n=|m|}^{N} x_n x_{n+m}, & m = -1, -2, \ldots \end{cases}$$

   This approximates the autocorrelation. In practice, we don't let m take infinitely many values, but instead look at it over a finite range of values from $-M$ to M for some integer M. The Power Spectral Density is computed through the Wiener Khinchin Theorem, using the DFT rather than the CTFT:

$$S_X(\omega) = \sum_{m=-M}^{M} R_X(m) e^{\frac{-j\omega m}{2M+1}}$$

   Using MATLAB, write a function that takes an input vector of N samples and an integer M as inputs, and returns the autocorrelation and PSD.

7. Use your function from above to plot the power spectral density white noise with some variance you choose so as to validate your function's operation. Make sure to use sufficiently large N and M.

   THERE IS A PROBLEM 8 DON'T STOP HERE

8. Use your function to plot the PSD of the of $X(t)$ from problem 4 with $\omega = 10000$ rad/s, as well as the PSD of the integral of $X(t)$. Please don't perform an integral numerically (except to check your work if you want to) – instead, use the relationship of PSDs at the input and output of an LTI system.