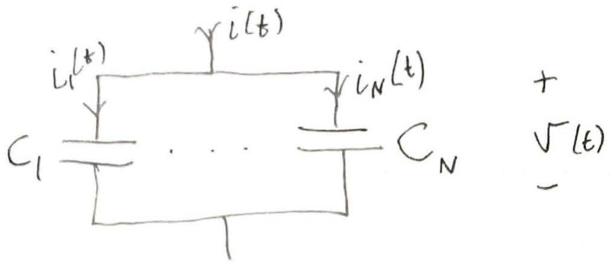
1.



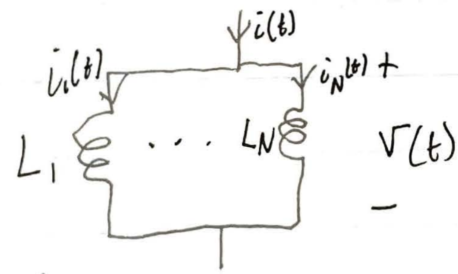
$$i(t) = i_1(t) + \dots + i_N(t)$$

$$C_k \frac{dv}{dt} = i_k \quad , \quad k = 1, 2, \dots, N$$

So $$i(t) = \sum_{k=1}^{N} i_k = \sum_{k=1}^{N} C_k \frac{dv}{dt} = \boxed{\left(\sum_{k=1}^{N} C_k\right) \frac{dv}{dt} = i(t)}$$

So $$C_{eq} = \sum_{k=1}^{N} C_k$$

2.



$$i(t) = \sum_{k=1}^{N} i_k(t) \quad , \quad L_k \frac{di_k}{dt} = v(t)$$

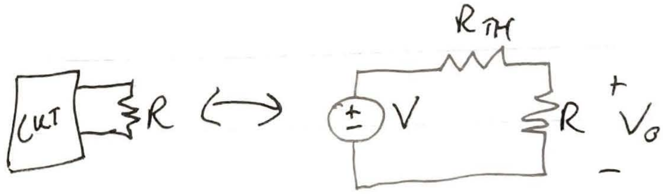
$$\frac{di}{dt} = \frac{d}{dt} \sum_{k=1}^{N} i_k(t) = \sum_{k=1}^{N} \frac{di_k}{dt} = \frac{v(t)}{L_{eq}} = \sum_{k=1}^{N} \frac{v(t)}{L_k}$$

So $$\frac{1}{L_{eq}} = \sum_{k=1}^{N} \frac{1}{L_k}$$

or

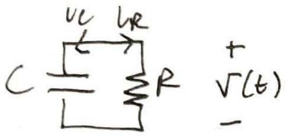
$$L_{eq} = \frac{1}{\sum_{k=1}^{N} 1/L_k}$$

3. $V_{TH} = V,$



$$V_o = V\frac{R}{R+R_{TH}} \Rightarrow R + R_{TH} = \frac{V}{V_o}R \Rightarrow R_{TH} = \left(\frac{V}{V_o} - 1\right)R$$

4.



$v(0) = V_0$

$i_R = -i_C$

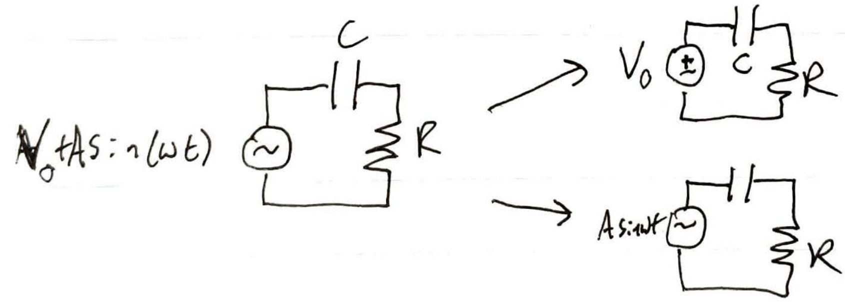
$\hookrightarrow \frac{v(t)}{R} = -C\frac{dv}{dt}$

So $\frac{-dt}{RC} = \frac{dv}{v(t)} \Rightarrow Ae^{-t/RC} = v(t)$, $v(0) = A = V_0$ so

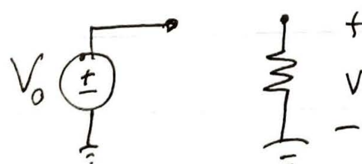
$$v(t) = V_0 e^{-t/RC}$$

5. As C grows large $i(t) = C\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{i(t)}{C} \approx 0$, so for short time intervals $\frac{dv}{dt} \approx 0 \Rightarrow v \approx$ constant or:
<u>Capacitor acts as a voltage source</u>

6.



7. At DC, C → O.C. so



$v = 0$, as no current can flow, the DC component has no effect

"The capacitor removes the DC component from the resistor's output"

8. >0: The current will grow towards $+\infty$
<0: The current will tend to $-\infty$
$=0$: The current will be bounded

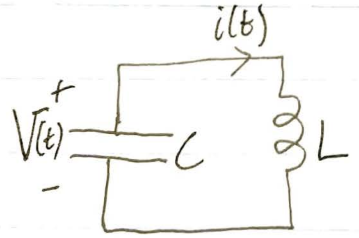
] inductors INTEGRATE Voltage

9. For a given two-terminal linear circuit, the circuit transfers power to some resistive load. This power is maximized when the load has resistance R_{TH}, the Thevenin resistance of the circuit.
This maximized power is given by $V_{TH}^2/4R_{TH}$

10.

	Pro	Con
overdamped	No oscillations	Slowest decay
Critically	fastest decay	unrealizable
Underdamped	faster decay than overdamped, realizable	oscillatory response

11.



$$L\frac{di}{dt} + \frac{1}{C}\int i(t)\,dt = 0 \quad \text{] KVL}$$

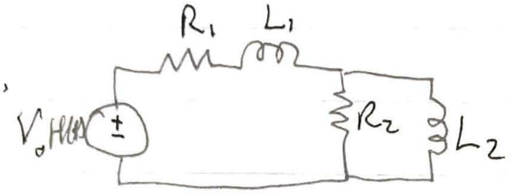
$$L\frac{d^2 i}{dt^2} + \frac{1}{C}i(t) = 0 \quad \text{] } \frac{d}{dt} \text{ both sides}$$

so Eqn: $\dfrac{d^2 i}{dt^2} + \dfrac{1}{LC}i(t) = 0$

Char. eqn: $m^2 + \frac{1}{LC} = 0 \Rightarrow m = \pm j\, 1/\sqrt{LC}$, $\omega = 1/\sqrt{LC}$, response

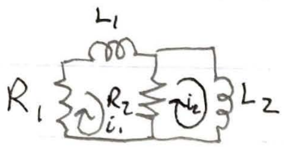
from $V_0 = V_0$ $\quad A\cos\omega t + B\sin\omega t = i(t)$, A, B not both zero as there is [energy initially in the system. Theoretically it does go forever. Realistically, there will be some parasitic resistance to cause damping.

12.



For transient, can turn off source

Mesh:



① $(i_1 - i_2)R_2 + i_1 R_1 + L_1 \frac{di_1}{dt} = 0$

② $L_2 \frac{di_2}{dt} + (i_2 - i_1)R_2 = 0$

From ②, $i_1 = \frac{L_2}{R_2}\frac{di_2}{dt} + i_2$. Plug into ①:

$$\left(\frac{L_2}{R_2}\frac{di_2}{dt} + \cancel{i_2} - \cancel{i_2}\right)R_2 + \left(\frac{L_2}{R_2}\frac{di_2}{dt} + i_2\right)R_1 + L_1 \frac{d}{dt}\left(\frac{L_2}{R_2}\frac{di_2}{dt} + i_2\right) = 0$$

$$\downarrow$$

$$\frac{L_1 L_2}{R_2}\frac{d^2 i_2}{dt^2} + \left(\frac{L_2 R_1}{R_2} + L_2 + L_1\right)\frac{di_2}{dt} + R_1 i_2 = 0$$

$$\downarrow$$

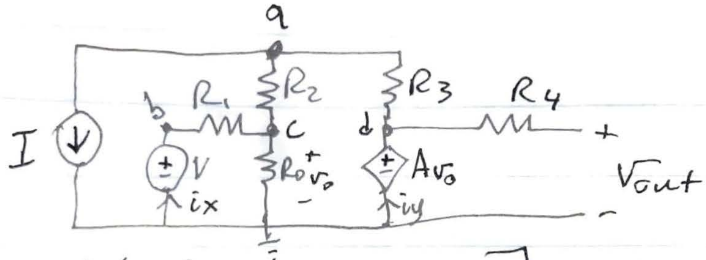
$$a\frac{d^2 i_2}{dt^2} + b\frac{di_2}{dt} + c i_2 = 0, \quad \alpha = \frac{b}{2a}, \quad \omega = \sqrt{\frac{c}{a}}$$

overdamped: $\alpha > \omega$

underdamped: $\alpha < \omega$

Critically damped: $\alpha = \omega$

13.



V_{TH}: $v_d = V_{TH}$, $v_o = v_c$] solve system, v_d will give V_{TH}

t_1: $v_b = V$

t_2: $Av_c = v_d$

a: $I + (v_a - v_c)G_2 + (v_a - v_d)G_3 = 0$

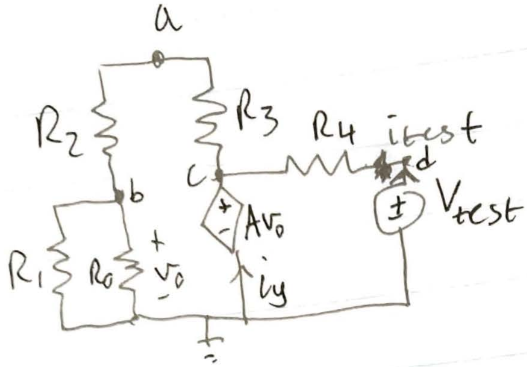
b: $i_x + (v_c - v_b)G_1 = 0$

c: $(v_b - v_c)G_1 + (v_a - v_c)G_2 = (v_c)G_o$

d: $i_y + (v_a - v_d)G_3 = 0$

$$\begin{pmatrix} G_2+G_3 & 0 & -G_2 & -G_3 & 0 & 0 \\ 0 & -G_1 & G_1 & 0 & 1 & 0 \\ G_2 & G_1 & -G_o-G_1-G_2 & 0 & 0 & 0 \\ G_3 & 0 & 0 & -G_3 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \\ v_d \\ i_x \\ i_y \end{pmatrix} = \begin{pmatrix} -I \\ 0 \\ 0 \\ 0 \\ V \\ 0 \end{pmatrix}$$

For R_{TH}:



$v_0 = v_b$

Solve system, $\frac{V_{test}}{i_{test}} = R_{TH}$

t_1: $A v_b = v_c$

t_2: $v_d = V_{test}$

a: $(v_a - v_b) G_2 + (v_a - v_c) G_3 = 0$

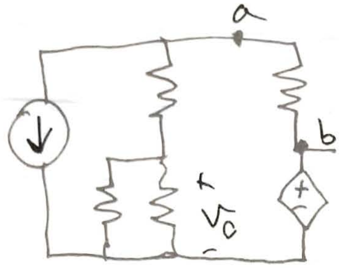
b: $(v_b - v_a) G_2 + v_b G_1 + v_b G_0 = 0$

c: $i_y + (v_a - v_c) G_3 + (v_d - v_c) G_4 = 0$

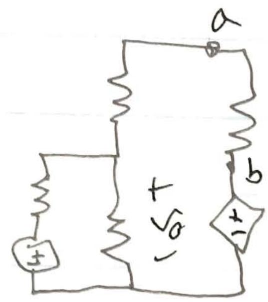
d: $i_{test} + (v_c - v_d) G_4 = 0$

$$\begin{pmatrix} G_2+G_3 & -G_2 & -G_3 & 0 & 0 & 0 \\ -G_2 & G_0+G_1+G_2 & 0 & 0 & 0 & 0 \\ G_3 & 0 & -G_3-G_4 & G_4 & 1 & 0 \\ 0 & 0 & G_4 & -G_4 & 0 & 1 \\ 0 & A & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \\ v_d \\ i_y \\ i_{test} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_t \end{pmatrix}$$

14.

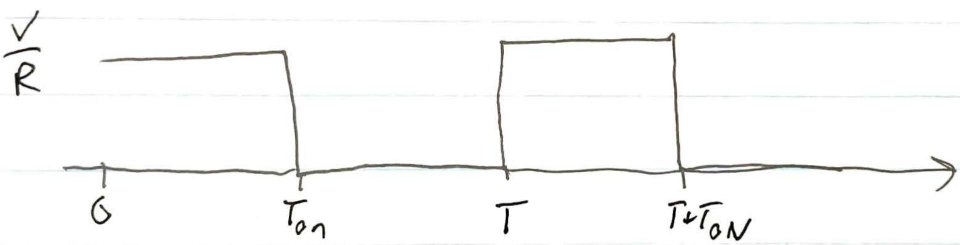
① 

and

② 

$V_1 = V_a - V_b$ for ckt 1 with MNA
$V_2 = V_a - V_b$ for ckt 2 with MNA

$V_1 + V_2 = V_{total}$

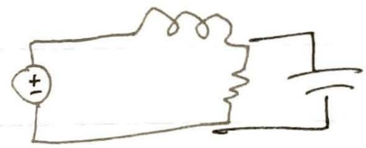
15. 

16. $P = \frac{1}{T}\int_0^T v(t)\,i(t)\,dt = \frac{1}{T}\int_0^{T_{ON}} V\left(\frac{V}{R}\right)dt + \frac{1}{T}\int 0\,dt$

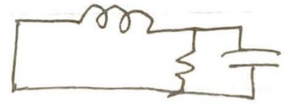
$= \frac{T_{ON}}{T}\frac{V^2}{R} = D\frac{V^2}{R}$

if switch always closed, power is $\frac{V^2}{R}$, so factor of D difference ($D<1$ so <u>less</u>)

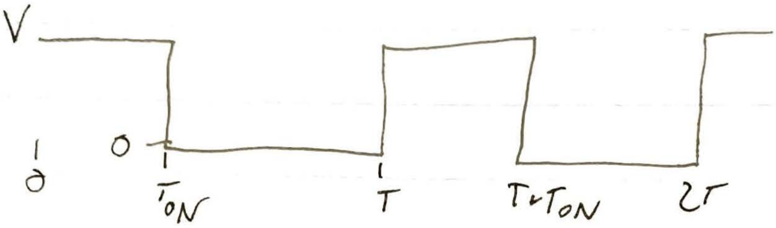
17. $0 \to T_{ON}$: $V_L + V_C = V$



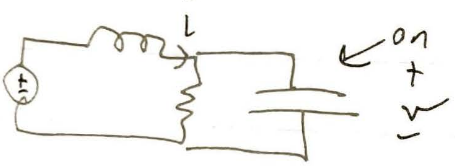
$T_{ON} \to T$: $V_L + V_C = 0$

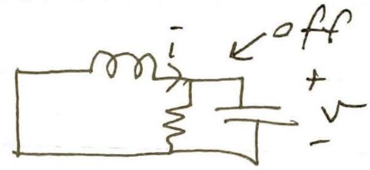


$V_L + V_C$:



18.





$i_{SS_{OFF}} = 0$

$V_{SS_{OFF}} = 0$

$i_{SS_{ON}} = V/R$

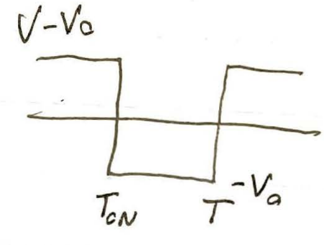
$V_{SSON} = V$

$v(T_{ON}-) = v(T_{ON}+)$ $\quad v(0) = v(T)$

$i(T_{ON}-) = i(T_{ON}+)$ $\quad i(0) = i(T)$

i.e. final conds. of <u>on</u> ckt are initial conds. of <u>off</u> ckt.

19. $V_L = \begin{cases} V - V_o, & 0 < t < T_{ON} \\ -V_o, & T_{ON} < t < T \end{cases}$



$L \frac{di}{dt} = V_L(t)$ $\quad i(t) = \frac{1}{L} \int V_L(t)\, dt = \begin{cases} \frac{1}{L}(V - V_o)t + C_0, & 0 < t < T_{ON} \\ \frac{1}{L}(-V_o)t + C_1, & T_{ON} < t < T \end{cases}$

$C_0 = I_o$, ~~I_{MAX}~~ $\quad \frac{1}{L}(V - V_o)T_{ON} + I_o = \frac{1}{L}(-V_o)T_{ON} + C_1 \Rightarrow C_1 = I_o + \frac{T_{ON}V}{L}$

↑

you need not actually do this to solve!

20. $\Delta i_{ON} = \left(\frac{1}{L}(V-V_o)T_{ON} + I_o - I_o\right) = \frac{1}{L}(V-V_o)T_{ON}$

$\cancel{\Delta i_{OFF} = \Delta i_{ON} =}$

$\Delta i_{OFF} = -\left(\frac{1}{L}(V-V_o)T_{ON} + I_o - \frac{1}{L}(-V_o)T - I_o - \frac{T_{ON}V}{L}\right)$

$= -\left(-\frac{V_o}{L}T_{ON} + \frac{V_o}{L}T\right) = \frac{V_o}{L}(T_{ON}-T)$

$\Delta i_{ON} = -\Delta i_{OFF}$

21. $\frac{V_o}{L}(T_{ON}-T) = -\frac{1}{L}(V-V_o)T_{ON}$

$\Rightarrow V_o(T_{ON}-T) = V_oT_{ON} - VT_{ON}$

$\Rightarrow -V_oT = -VT_{ON}$

$\Rightarrow \frac{V_o}{V} = \frac{T_{ON}}{T} = D$

22. C large, for example