HW REVIEW

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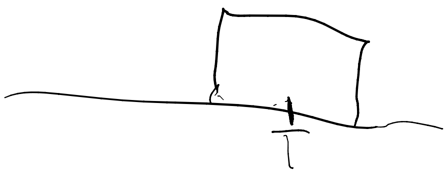
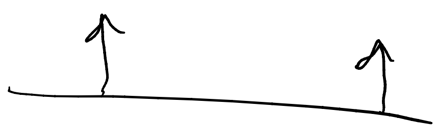
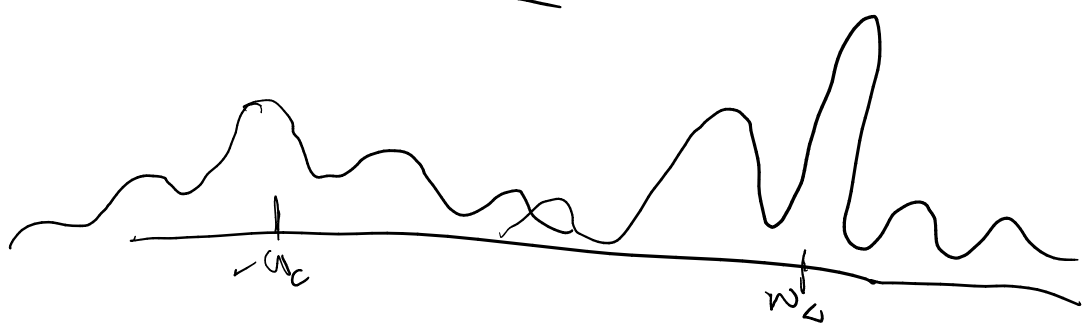


$\times$

$\cos(\omega_0 t)$

$e^{-j\omega T}$





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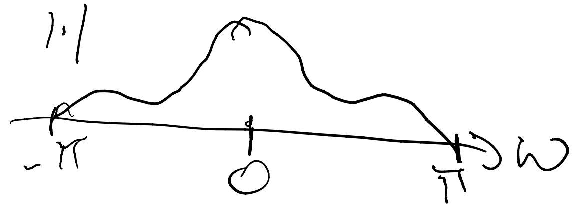
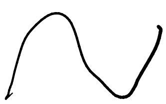
FFT = $\underbrace{\text{fast}}$ FT

→ DFT = Discrete Fourier Transform

DTFT: $\sum_n x[n] e^{-j\omega n} \in \mathcal{F}(\mathbb{R}, \mathbb{C})$



 sin(x) → rep. as vector

DFT = sampled DTFT

$x(t) \longrightarrow x[n]$ Matlab vector approx. $x(t)$

$$X(\Omega) = \int x(t) e^{-j\omega t} dt \approx \sum x[n] e^{-j\omega n} \Delta t$$

$$= \Delta t \, X(\omega)$$

$$\approx \Delta t \, DFT\{x\}$$

$$\approx \underbrace{\Delta t}\, DFT\{x\}$$

$$\Delta t = 1/f_s$$

AM

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Setting

Some signal $m(t)$ baseband, $m(t) \in \mathbb{R}$

assume finite bandwidth (BW), W

Hermitian

Ex. m audio, then $W \approx 15\text{kHz}$

- Not at RF, so can't transmit across channel
- I want many channels, but a bunch of these would interfere

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Modulation THM we want to "lift" the audio signal to a higher freq.

Carrier Signal $c(t) = A_c \cos(\omega_c t)$, $\omega_c = 2\pi f_c$

$m(t)c(t)$? maybe...

DSB-SC AM

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D) Double Sideband Suppressed Carrier Amplitude modulation

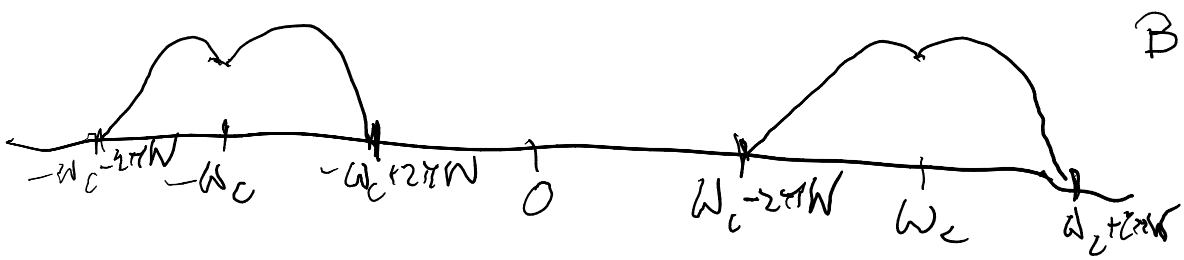
$$u(t) = m(t)c(t) = A_c m(t)\cos(w_c t)$$

$$= A_c m(t)\left(\frac{e^{jw_c t} + e^{-jw_c t}}{2}\right)$$

$\downarrow$

$$U(w) = \frac{A_c}{2}\left(M(w - w_c) + M(w + w_c)\right)$$

as long as $w_c > 2\pi W$



BW 2W

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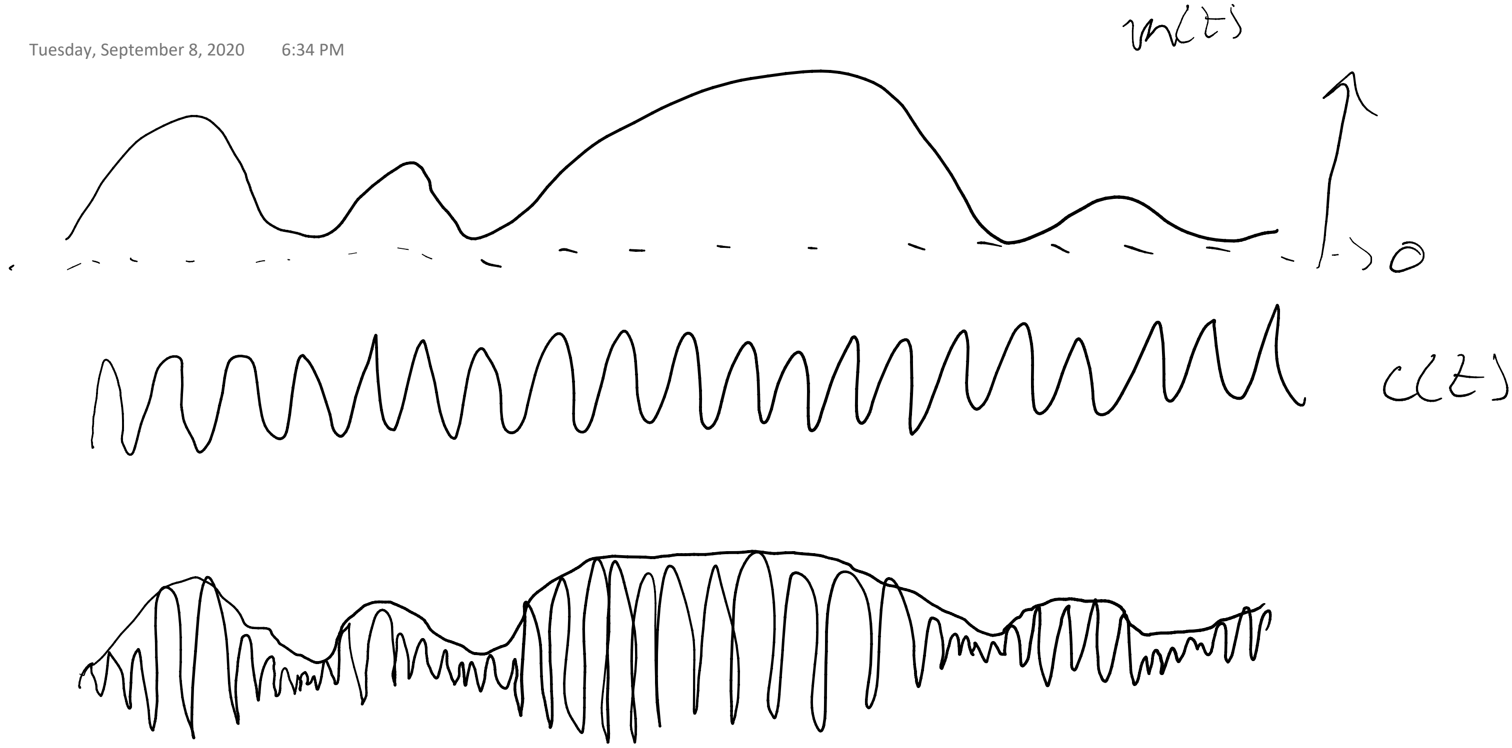

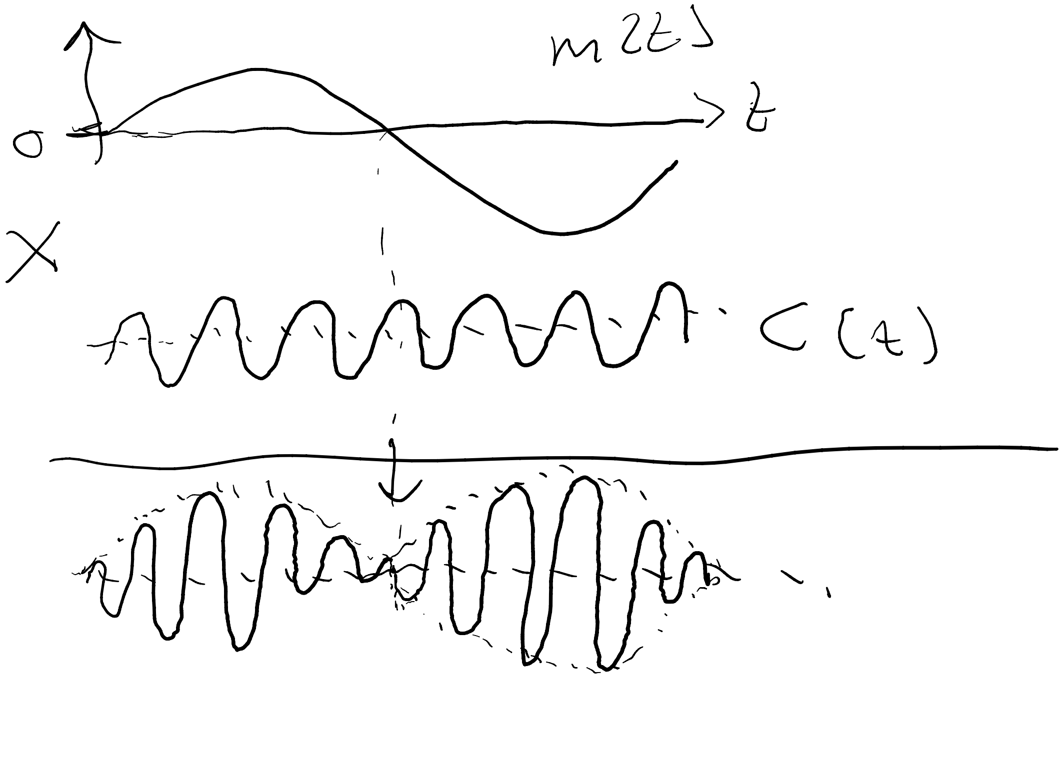

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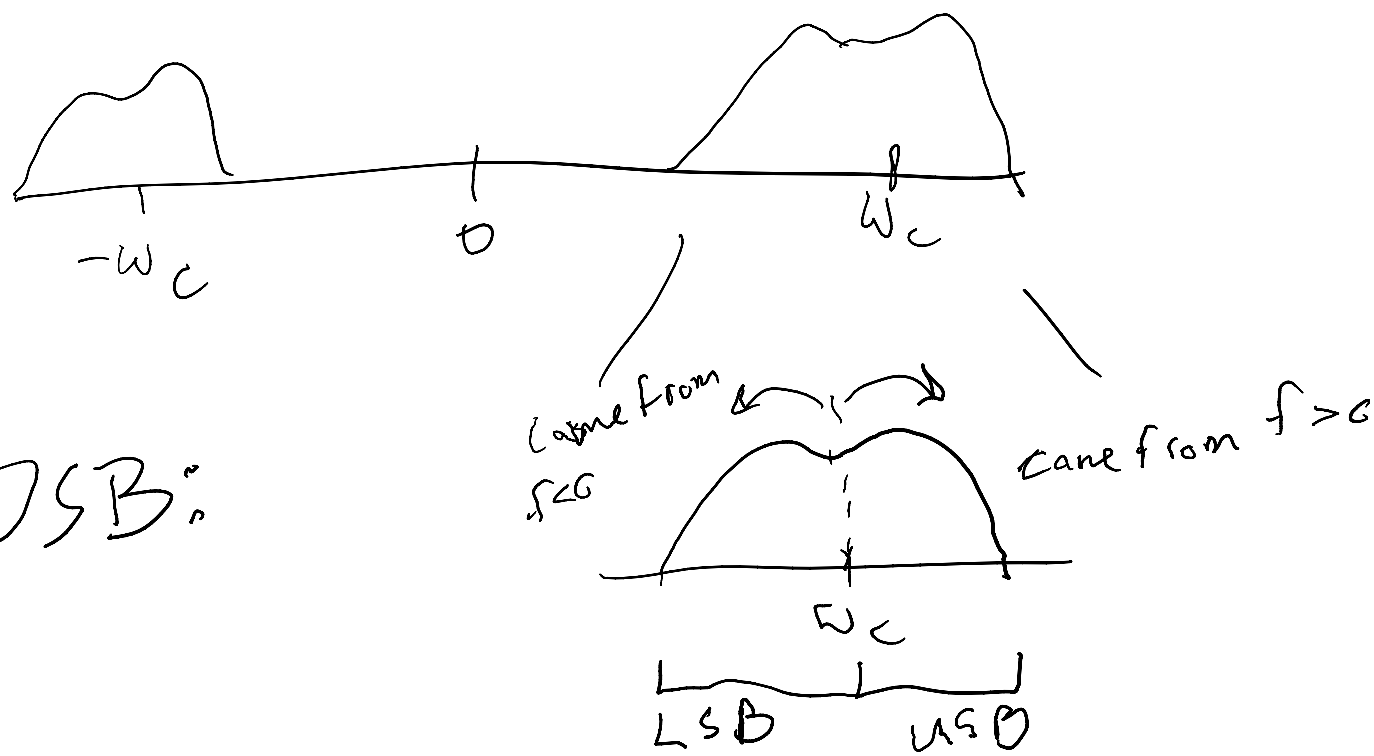
Why is it called DSB-SC?



DSB:

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$$P_u = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A_c^2\, m^2(t)\cos^2(\omega_c t)\,dt$$

$$= \lim \frac{1}{T}\int \frac{A_c^2}{2} m^2(t)\left(1+\cos(2\omega_c t)\right)dt$$

$$= \lim_{T\to\infty}\frac{A_c^2}{2T}\left(\int_{-T/2}^{T/2} m^2(t)\,dt + \underbrace{\int_{-T/2}^{T/2} m^2(t)\cos(2\omega_c t)\,dt}_{0}\right)$$

$$= \frac{A_c^2}{2}P_m$$

No power coming from the "carrier component"

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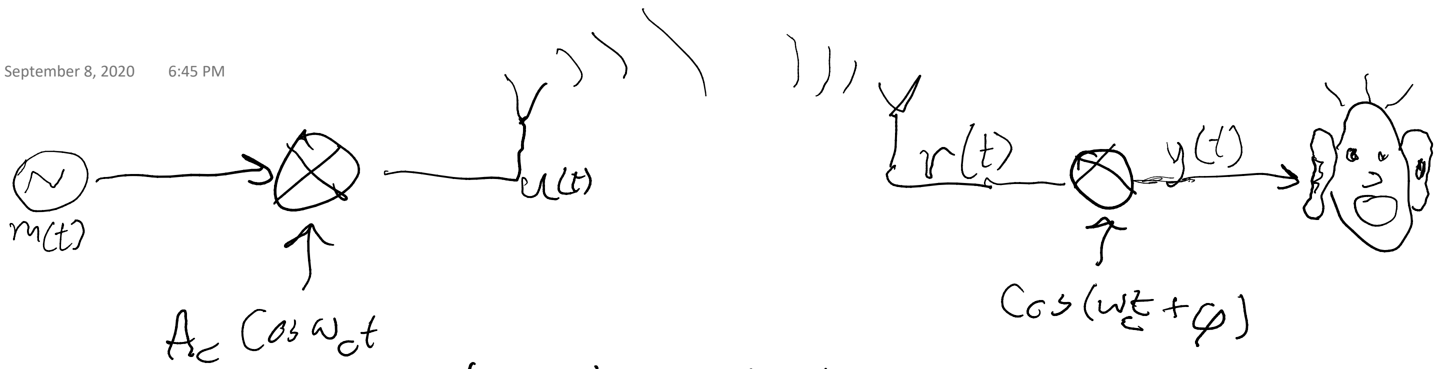


<u>No noise scenario</u>

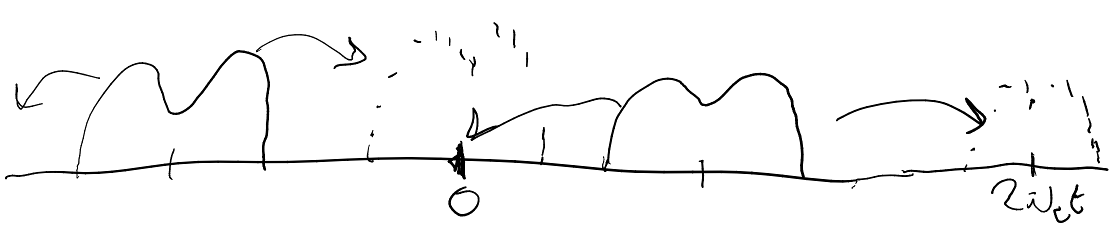
$$r(t) = u(t - \Delta t)$$

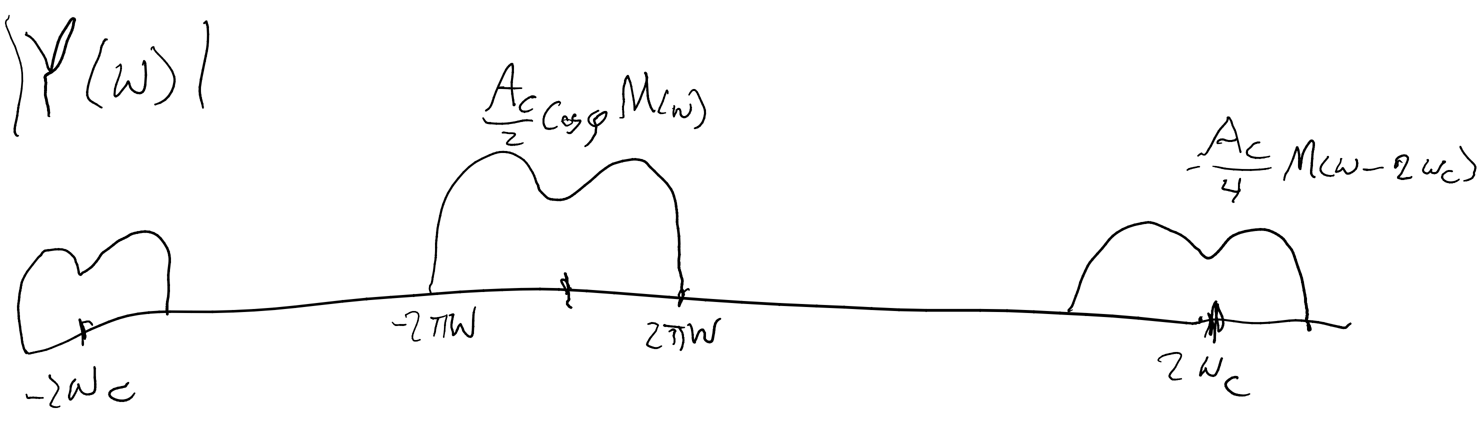
First, assume $\Delta t = 0$

$$r(t) = u(t) = m(t)\, c(t)$$

$$y(t) = A_c m(t) \cos(\omega_c t) \cos(\omega_c t + \varphi)$$

$$= \frac{A_c}{2} m(t) \cos\varphi + \frac{A_c}{2} m(t) \cos(2\omega_c t + \varphi)$$





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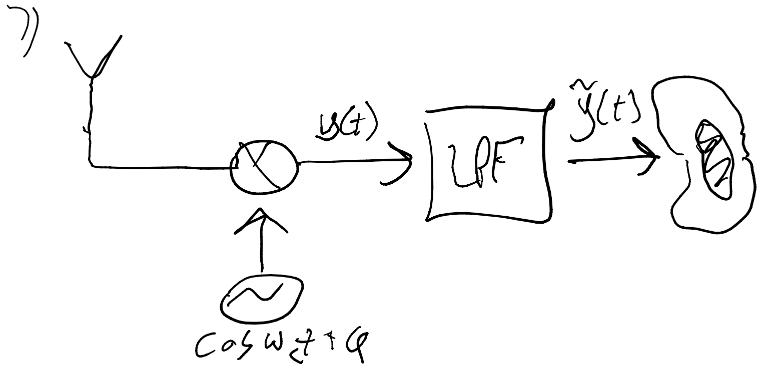

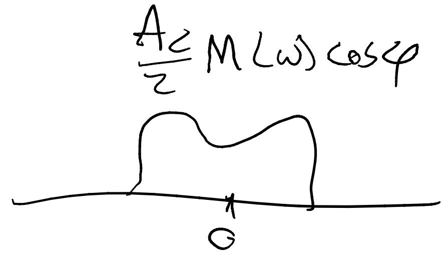

$$\tilde{y}(t) = \frac{A_c}{2} m(t) \cos\varphi$$

The power received is at worst $\underline{\underline{0}}$ and dep. on phase

PHASE can make or break a comm scheme

DSB-SC is unpopular due to phase sensitivity.

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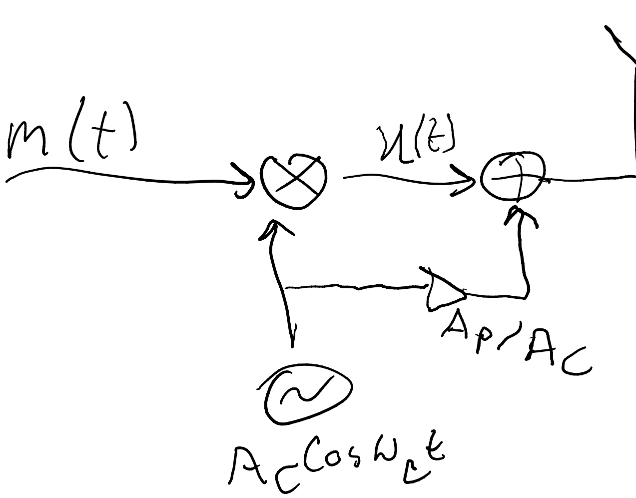
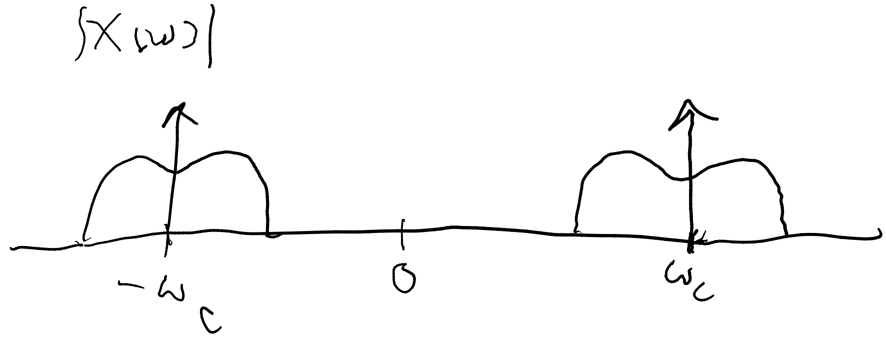
An Rx which "knows about" the carrier phase is called either "coherent" or "synchronous" (phase-coherent)

<u>Solution</u> Add a "pilot tone", which is a copy of the carrier on top of the message



$$x(t) = u(t) + \underbrace{A_p \cos \omega_c t}_{\text{pilot tone}}$$



"DSB AM"

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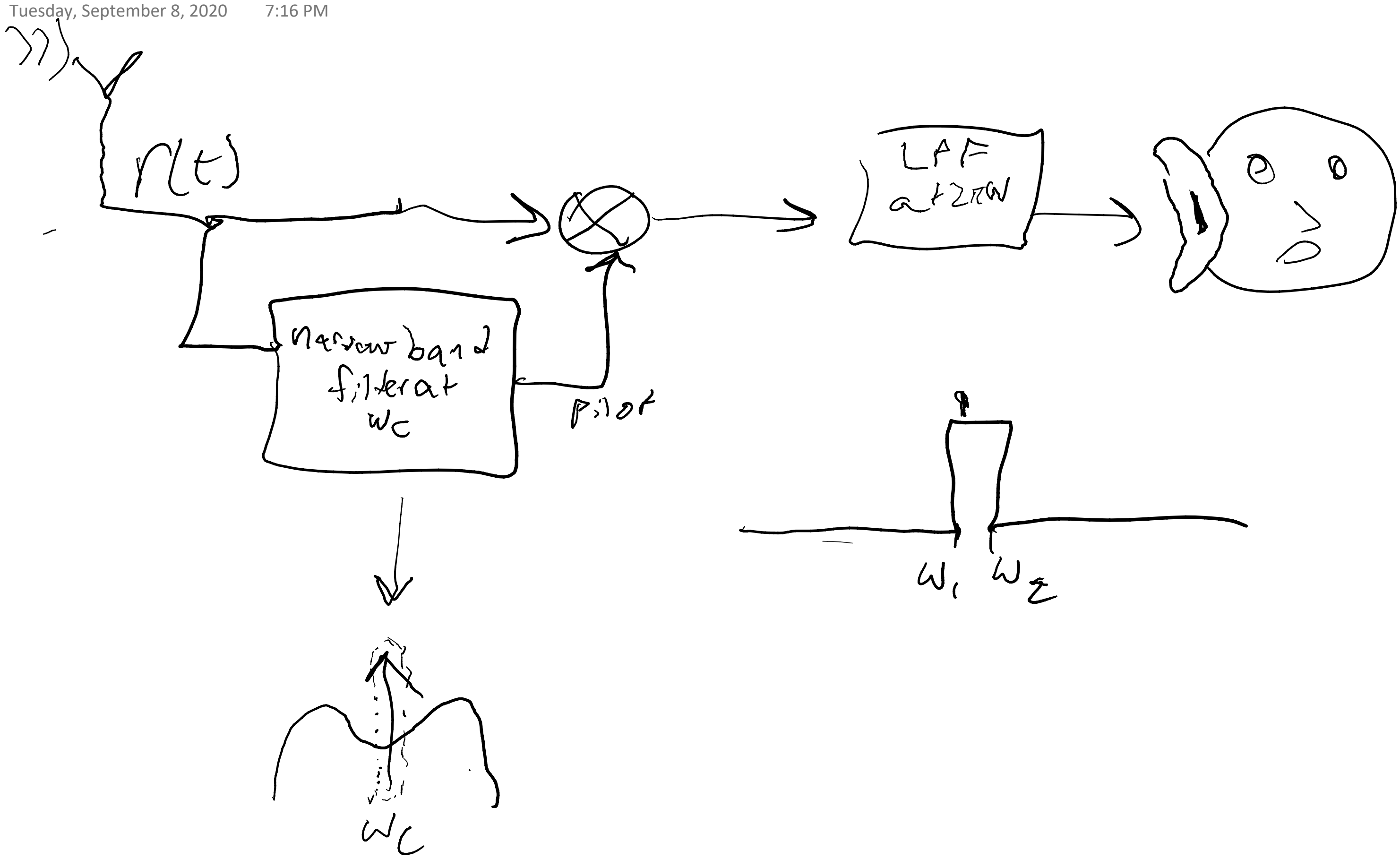

Conventional AM

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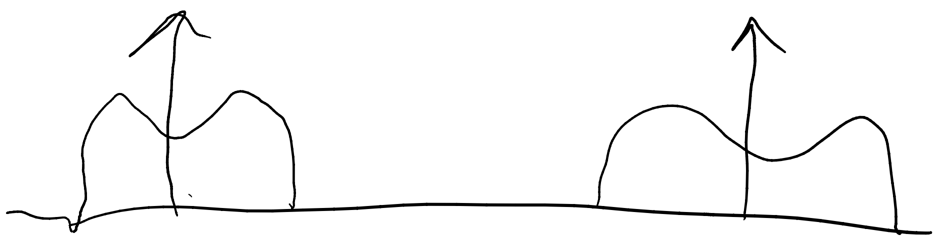
assume $|m(t)| \leq 1 \ \forall t$

$$u(t) = A_c \left(1 + m(t)\right) \cos \omega_c t$$



this is a DSB AM scene

$1 + m(t) \geq 0$

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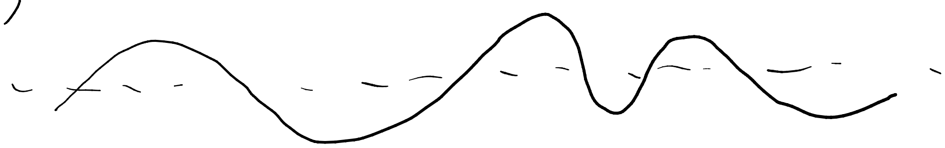
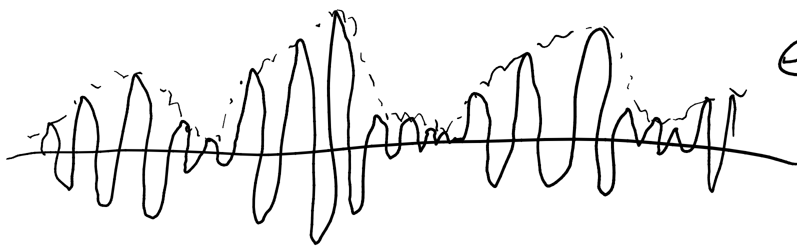
$m(t)$



$1+m(t)$

$u(t)$



envelope is like $m(t)+1$

generally in DSB, envelope is like $|m(t)|$

here, $|1+m(t)| = 1+m(t)$

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Convention: $a = \max(|m(t)|)$

write $m(t) = a m_n(t)$ so $\max|m_n(t)| = 1$

if $0 < a \leq 1$, then safe

if $a > 1$, then "overmodulating"

Call a the "modulation index"

$$P_u = \lim_{T\to\infty} \int_{-T/2}^{T/2} (1 + a m_n(t))^2 A_c^2 \cos^2(\omega_c t)\, dt$$

$$P_m = \lim \int (1 + a m_n(t))^2\, dt = 1 + a^2 P_{m_n}$$

$$P_u = \underbrace{\frac{A_c^2}{2}}_{\sim \text{power in carrier}} + \underbrace{\frac{A_c^2}{2} a^2 P_{m_n}}_{\text{would be power in DSB-SC}}$$

larger power req. than DSB-SC by power in carrier

"Cost" of sending a carrier component

a lot of power, no info

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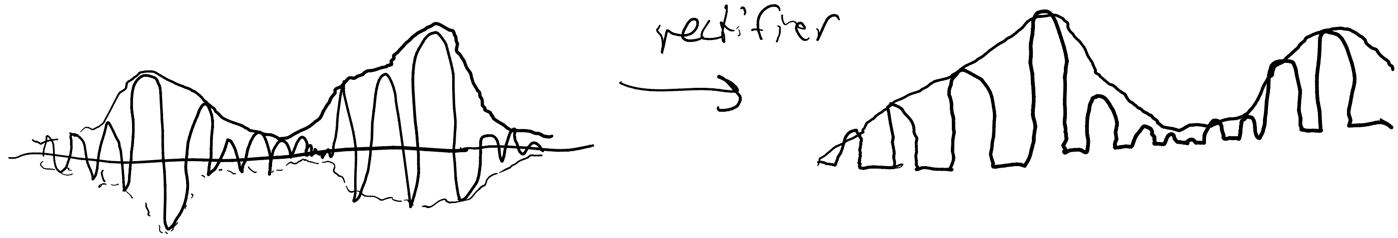
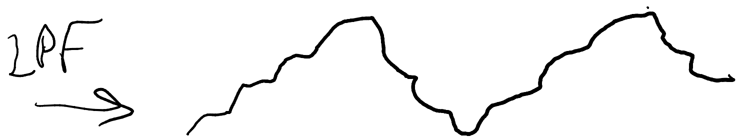
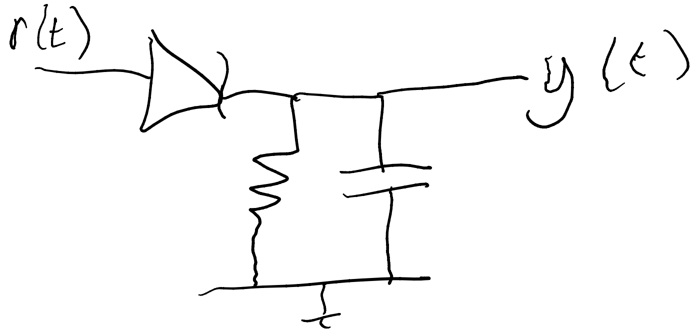
Conventional used entirely due to simple demod





"Close" to the message
significant distortion



CHEAP AS HELL

Conventional AM has simplest demod implementation

good if many RX

Worse in terms of signal quality

less power efficient

not BW-efficient

SSB AM

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Single sideband AM

BW = W compared to DSB's 2W

$|V(\omega)|$

USSB AM

or

LSSBAM

This is the most BW-efficient possible

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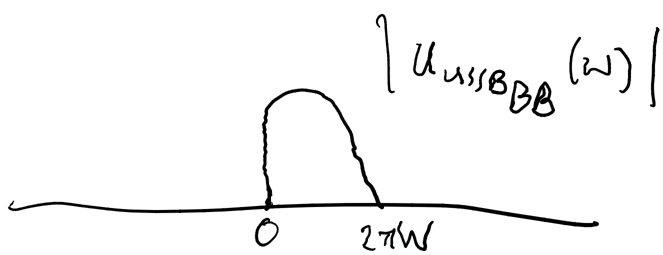
Hilbert Transform

$$H(\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}$$

Look at U at baseband

$u_{ssB_{BB}} = \tilde{U}(\omega)$



$$\tilde{u}(t) = I(t) + j\,Q(t)$$

$$u(t) = I(t)\cos\omega_c t - Q(t)\sin\omega_c t$$

$$M(\omega) = \begin{cases} \tilde{U}(\omega), & \omega > 0 \\ \tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

So $$H(\omega)M(\omega) = \begin{cases} -j\tilde{U}(\omega), & \omega > 0 \\ j\tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

$$= \hat{M}(\omega)$$

$$j\hat{M}(\omega) = \begin{cases} \tilde{U}(\omega), & \omega > 0 \\ -j\tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

$$j\hat{M}(\omega) = \begin{cases} \tilde{U}(\omega), & \omega > 0 \\ -\tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

$$M(\omega) + j\hat{M}(\omega) = \begin{cases} 2\tilde{U}(\omega), & \omega > 0 \\ 0, & \omega < 0 \end{cases}$$

$$= 2\tilde{U}(\omega)$$

$$\tilde{U}(\omega) = \frac{M(\omega)}{2} + j\frac{\hat{M}(\omega)}{2}$$

$$\downarrow$$

$$\tilde{u}(t) = \frac{m(t)}{2} + j\frac{\hat{m}(t)}{2} = I(t) + jQ(t)$$

$$I(t) = \frac{m(t)}{2}, \quad Q(t) = \frac{\hat{m}(t)}{2}$$

$$u(t) = \frac{m(t)}{2}\cos\omega_c t - \frac{\hat{m}(t)}{2}\sin\omega_c t$$

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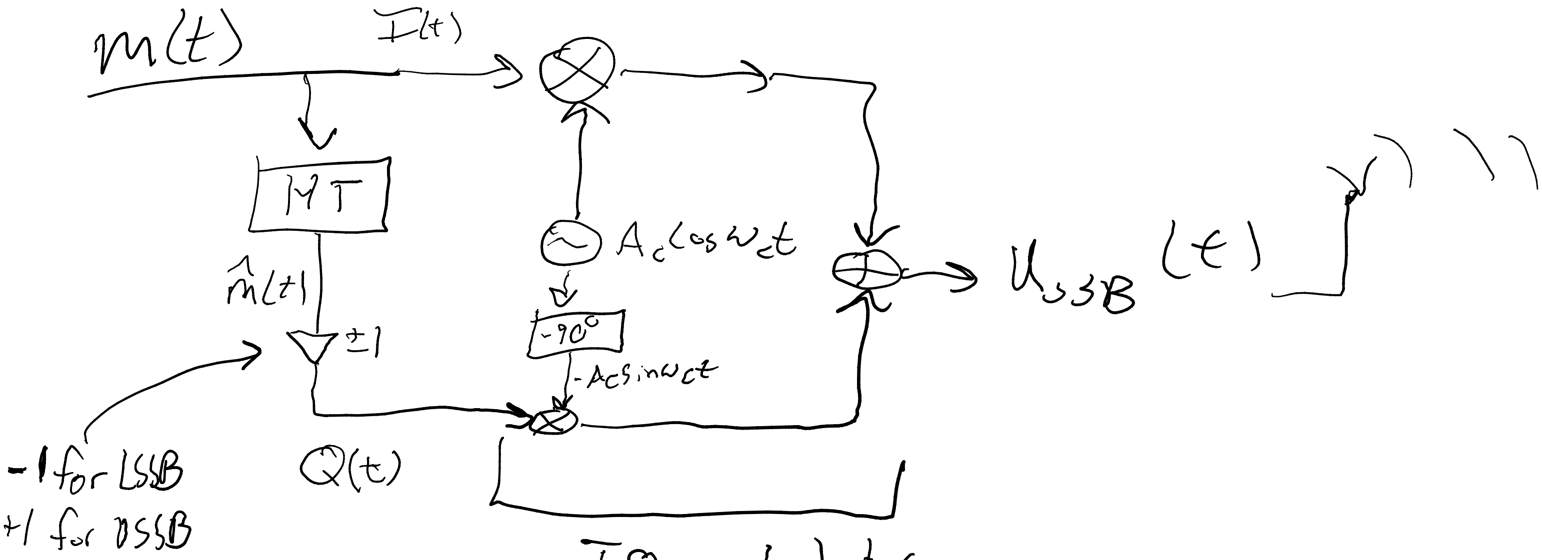
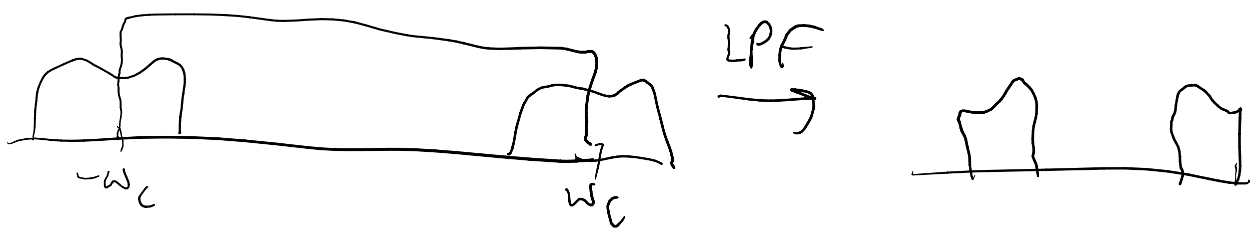
$$I_{LSSB} = I_{USSB} = \frac{m(t)}{2}$$

$$Q_{LSSB} = -Q_{USSB} = \frac{-\hat{m}(t)}{2}$$

$$\boxed{\begin{aligned} u_{LSSB}(t) &= A_c m(t) \cos w_c t + A_c \hat{m}(t) \sin w_c t \\ u_{USSB}(t) &= A_c m(t) \cos w_c t - A_c \hat{m}(t) \sin w_c t \end{aligned}}$$

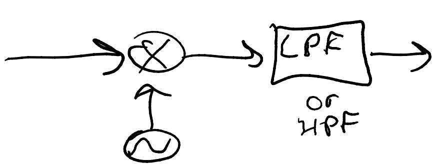


IQ modulator
or a quadrature modulator



alternate method uses LPF (LSSB)
or
HPF (USSB)

but requires really tight filter



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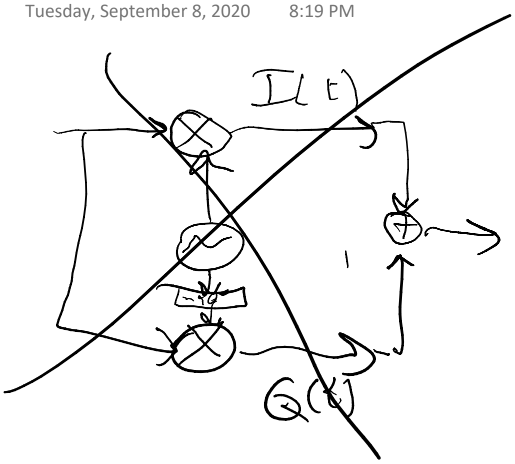
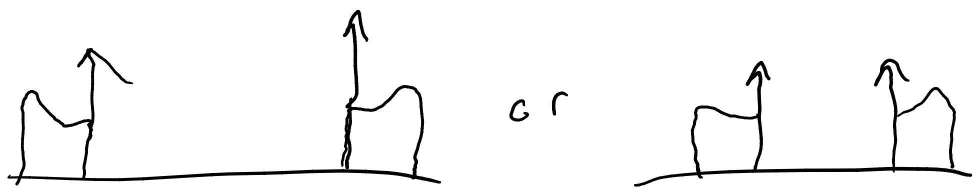


need phase coherence

so send pilot tone

$x_{SSB}(t) + A_p \cos \omega_c t$ transmitted



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SSB is more BW efficient than DSB, than conventional

not necessarily power efficient compared to an SC method

X doesn't matter

So actually, SSB is pretty power efficient - transmits ~half the power in the message that DSB does

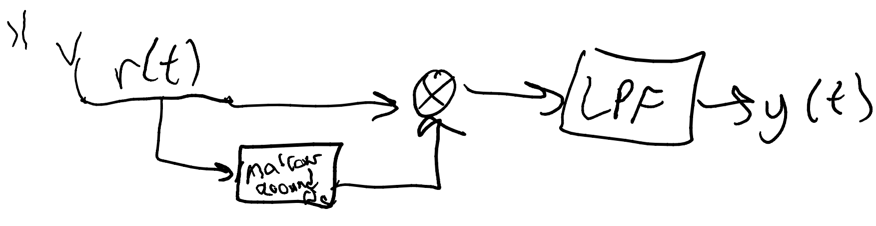
$$DSB \sim P_m + P_c$$

$$SSB \sim \frac{1}{2}P_m + P_c > \frac{1}{2}P_{DSB}$$

$$< P_{DSB}$$

more BW efficient, power efficient <u>but</u> complex TX <u>and</u> can't just use a rectifier RX

as an RX, just the same as DSB (not conv)



Vestigial Sideband AM

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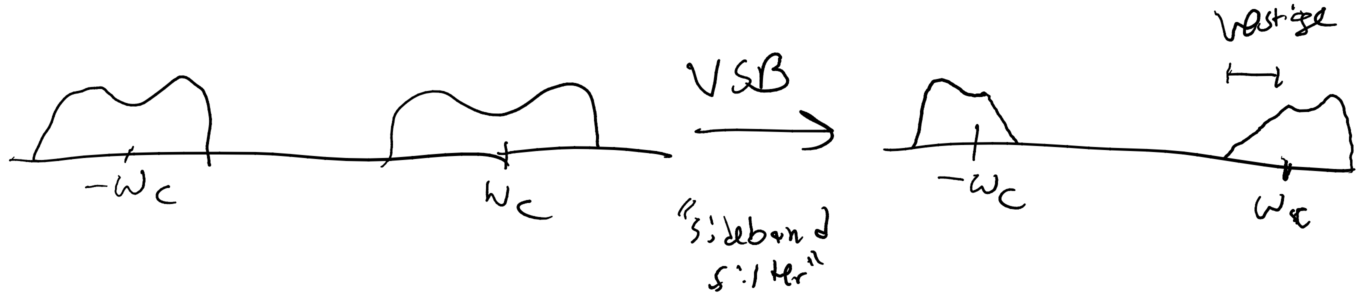
VSBAM

like SSB, but allows "vestige" of the other sideband



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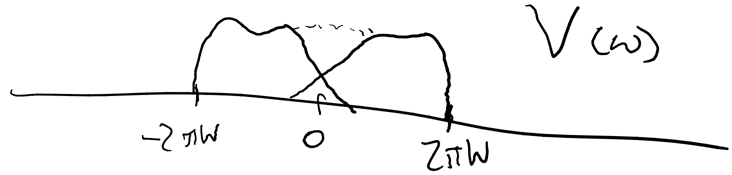
What you want to happen is when you demod, you want vestiges to give you the "right signal"

after $\otimes \cos\omega_c t$



, say sideband filter is $H(\omega)$

$$V(\omega) = \big(M(\omega) H(\omega - \omega_c) + M(\omega) H(\omega + \omega_c) \big) A$$

$$\propto M(\omega)$$

$$H(\omega - \omega_c) + H(\omega + \omega_c) = C \quad \text{for } |\omega| \le 2\pi W$$

H is flat in non overlapping range" $\omega_c < |\omega| < \omega_c + \omega_a$

linear symm. in overlap

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Vestigial "realistic" imp. of the simpler SSB scheme w/ filter

in simpler SSB scheme only viable if signal has no low-freq. components

- good enough for music
- <u>bad</u> for TV

Vestigial works even for signals w/ large low-freq. components

More BW / Power efficient than DSB
less so than SSB

and so it's kind of application specific

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SSB - best on power and BW and no rect. distortion

Conv - cheap RX

VSB - good where SSB fails to be simple (large low freq. comps)

DSB-SC - BAD but natural pedagogically

FDM

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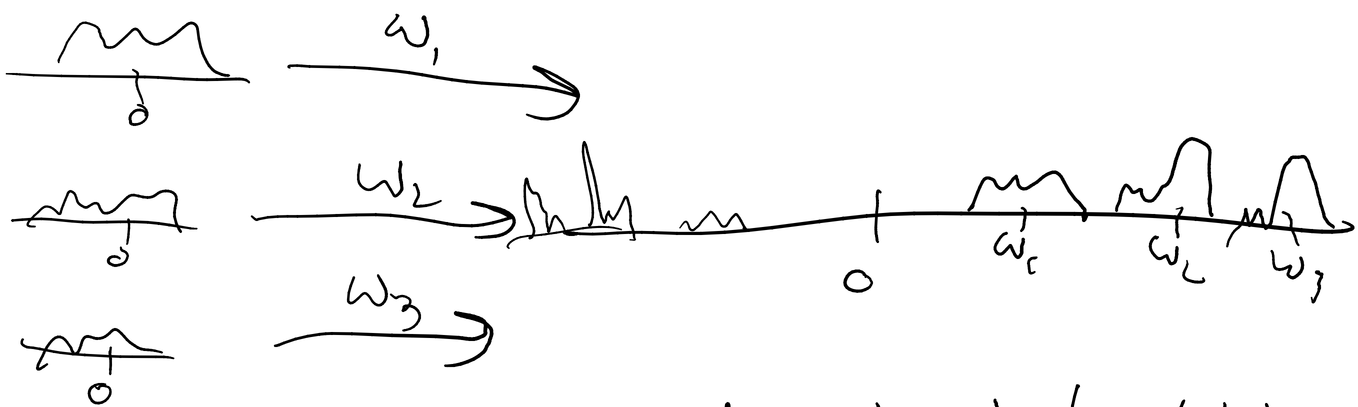
Frequency Division Multiplexing

means by which we send multiple AM signals at once



signals overlap at base bands but don't interfere at bandpass.