Communication Theory Homework 4

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Fall 2020

1. I have a baseband analog signal with a 20 kHz bandwidth. How fast must I sample to ensure a 4 kHz guard band?

2. Suppose I have bandlimited noise with PSD 8 for frequency less than 200 kHz in absolute value. Sampling this noise at Nyquist and applying a 16-level quantizer, what are the rate and distortion (mean squared error)? What is the SQNR?

   The next few problems will consider the following setup: Suppose I am doing a digital logic design project, and I represent 1 as a 2.5 V pulse of duration A and 0 as a -2.5 V pulse of duration A. My chips perform a *least squares* decision when they receive a signal. The noise over a wire is largely thermal noise caused by statistical variations in charge carriers, modeled as AWGN. This noise has variance $4k_BTR$ where $k_B \approx 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, T is the absolute temperature (in Kelvin) and R is the total resistance of the wire. A commonly used wire type (available in the lab at school) is 22 AWG solid copper wire, for which 1 m of wire has 52.7 mΩ resistance. Recall that resistance is proportional to length.

3. Describe the operation of a least squares decision system for this problem.

4. Is the least squares decision rule the same as a matched filter decision rule in this case? Is it the same as maximum likelihood? Is it the same as maximum *a posteriori*? If there is not enough information to answer, give the conditions that would guarantee the equality. In each case, don't just say "yes" or "no", provide a one-sentence explanation.

5. What is the name of this modulation scheme? What is the basis? Draw the geometrical representation.

6. For this scheme, suppose I transmit a 1 75% of the time – what are the MAP decision regions?

7. Assume now that the symbols are equiprobable. In terms of A, T and wire length L, write a formula for the SNR and probability of error.

8. Assume we are operating the system at room temperature – how long does the wire have to be to yield an error probability of 1/10? Write this number in lightyears. Compare this to the size of the Milky Way. This should give you an appreciation for how robust wired communication is to thermal noise.
   "Professor, my project doesn't work, and I think it might be because of thermal noise!"